Quasiparticle states in heavy nuclei

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- Study of structure of heaviest nuclei to understand the mechanism of their formation in fusion reactions

- Identification of heavy nuclei by $\alpha$-decay needs the analysis of isomer states

- Population of isomer states in reactions

- Search of shells and subshells closure
\[ \beta_i = \frac{a_i}{b_i} \]

\[ \beta_1 = \beta_2, \text{ even multipolarities; } \quad \beta_1 \neq \beta_2, \text{ odd and even multipolarities} \]

\( R_0 \) is the radius of spherical nucleus

Parametrisation of nuclear shape with TCSM
\[ H = T + V(\rho, z) + V_{LS} + V_L^2 \]

\[
\frac{1}{2} m \omega_z^2 (z - z_1)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, \ z < z_1
\]

\[ V(\rho, z) = \begin{cases} \\
\frac{1}{2} m \omega_\rho^2 \rho^2, \ z_1 < z < z_2 \\
\frac{1}{2} m \omega_z^2 (z - z_2)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, \ z > z_2
\end{cases} \]

\[
V_{LS} = -\frac{2 \hbar \kappa_i}{m \omega_0 i} (\nabla V \times \vec{p}) \hat{s}
\]

\[
V_L^2 = -\hbar \omega_0 i \kappa_i \mu_i \hat{l}^2 + \hbar \kappa_i \mu_i \omega_0 i N_1 (N_1 + 3)/2 \delta_{if}
\]

\[
\omega_0 i = 41 \text{ MeV} / A_i^{1/3}, \quad A_i = a_i b_i^2 / 1.22^3, \quad \omega_\rho / \omega_z = a_i / b_i, \quad z_2 - z_1 = 2R_0 \lambda - a_1 - a_2
\]
Comparison with other calculations

$^{248}$Fm gs.:

$\lambda = 1.18, \beta = 1.28 \rightarrow \beta_2 = 0.25, \beta_4 = 0.027$

P. Möller et al. $\beta_2 = 0.235, \beta_4 = 0.049$

For $^{247,248,249}$Fm, the microscopic corrections are -3.85, -3.88, and -4.3 MeV.

P. Möller et al.: -3.52, -3.57, and -3.97 MeV

The values of $\Delta$ differ within 0.1 MeV.
• TCSM

△ A. Sobieszewski et al.

○ P. Möller et al.

◇ S. Goriely et al.

N=155
Potential energy

\[ U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta) \]

Binding energy

\[ B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v (1 - 1.78 \left(\frac{N - Z}{A}\right)^2) A + ... \]
\[ a_v = 15.83 \text{ MeV} \]

\[ Q_\alpha \text{ energy} \]

\[ Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z - 2, A - 4) \]

Alpha decay half-lives \( T_\alpha \) (A. Sobczewski et al.)

\[ \log_{10} T_\alpha(Z, A) = 1.5372 Z Q_\alpha^{-1/2} - 0.1607 Z - 36.573 \]
Strength parameters of paring interaction

\[ G_n = (19.2 \pm 7.4 \frac{N-Z}{A}) A^{-1} \text{MeV} \]

\[ A \approx 250 \rightarrow G_n \approx 0.075 \text{MeV}, \ G_p \approx 0.085 \text{MeV} \]

One-quasiparticle excitations

\[ E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} - \sqrt{(e_\mu' - e_F)^2 + \Delta^2} \]

Two-quasiparticle excitations

\[ E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} + \sqrt{(e_\mu' - e_F)^2 + \Delta^2} \]
N=149 isotones

QPM

\[ E_{\mu} (\text{MeV}) \]

\[ 1/2^+ [620] \]

\[ 7/2^+ [613] \]

\[ 5/2^+ [622] \]

\[ 7/2^- [743] \]

\[ 1/2^+ [631] \]

\[ 9/2^- [734] \]

\[ 7/2^+ [624] \]
$N=149$ isotones

SLy4

$E_\mu$ (MeV)

$\frac{7}{2}^{+}[620]$  
$\frac{1}{2}^{-}[770]$  
$\frac{3}{2}^{+}[622]$  
$\frac{7}{2}^{+}[613]$  
$\frac{1}{2}^{+}[631]$  
$\frac{9}{2}^{-}[734]$  
$\frac{7}{2}^{-}[743]$  
$\frac{5}{2}^{+}[622]$  
$\frac{7}{2}^{+}[624]$
The smooth change of energies of almost all one-quasiparticle states in the isotone chain.

The deformation parameters of the $N=149$ nuclei treated are almost the same. Although different methods of calculations give various deformations of the ground state. For example, in the case of $^{251}$No $\beta_2=0.234$ and $\beta_4=0.057$ are resulted from the TCSM, $\beta_2=0.296$ and $\beta_4=0.01$ from the HB with Sly4 parameterization.

Long-living isomer in $^{251}$No: $1/2^+[631]$, about 1 s
For the $N=151$ nuclei, all our calculations with different methods give the $1/2^+[620]$ states below the $5/2^+[622]$ states. The $7/2^+[624]$ state is expected below the $5/2^+[622]$ state. The $1/2^+[620]$ state goes down in heavier nuclei and could become the isomer state in $^{255}$Rf.

In the $N=149$ and 151 nuclei the $1/2^+[620]$ single-particle state is above and the $5/2^+[622]$ and $1/2^+[631]$ single-particle states are below the corresponding Fermi levels. These levels go down with deformation. If one adjusts in $^{251}$No the $1/2^+[631]$ level closer to the Fermi level than the $5/2^+[622]$ level, then in $^{253}$No the $1/2^+[631]$ level would be below the $5/2^+[622]$ level as well. Thus, one can not simultaneously adjust the calculated one-quasiparticle states to the experimental assignments for $^{251}$No and $^{253}$No. This statement is true if the deformations of these nuclei are close as follows from our calculations.
\[ \begin{align*}
5/2^+[633] & \quad 5/2^+[503] \\
1/2^+[501] & \quad 5/2^-[752] \\
5/2^-[752] & \quad 9/2^+[734] \\
9/2^+[734] & \quad 5/2^-[622] \\
7/2^+[624] & \quad 1/2^+[631] \\
7/2^+[631] & \quad 7/2^-[743] \\
5/2^-[743] & \quad 9/2^+[734] \\
7/2^+[743] & \quad 1/2^+[624] \\
5/2^+[622] & \quad 1/2^+[620] \\
7/2^+[622] & \quad 8.82 \quad 7/2^+[624] \\
5/2^+[624] & \quad 7/2^+[624] \quad 8.64 \\
7/2^+[624] & \quad 5/2^+[622] \quad 7.88 \\
5/2^+[622] & \quad 7/2^+[623] \quad \text{exp.} \quad 7.95 \\
7/2^+[623] & \quad 243 \text{Cf} \\
1/2^+[623] & \quad 247 \text{Fm} \\
5/2^+[622] & \quad 251 \text{No} \\
7/2^+[624] & \quad 255 \text{Rf} \\
9/2^+[734] & \quad 8.75 \quad \text{MeV} \\
8.82 & \quad 8.85 \text{MeV}
\end{align*} \]
Rotational bands

The prediction of alternating parity bands at energy larger than 0.5 MeV.
N=153 isotones

QPM

E (MeV)

$\mu$

$5/2^+ [622]$

$1/2^+ [770]$

$9/2^+ [615]$

$11/2^+ [725]$

$7/2^+ [624]$

$9/2^+ [734]$

$3/2^+ [622]$

$7/2^+ [613]$

$1/2^+ [620]$

$^{247}\text{Pu}$ $^{249}\text{Cm}$ $^{251}\text{Cf}$ $^{253}\text{Fm}$ $^{255}\text{No}$ $^{257}\text{Rf}$ $^{259}\text{Sg}$
The $7/2^+[613]$ state is expected to be the isomer at least for $N=153$ nuclei up to $^{255}$No. In $^{257}$Rf and $^{259}$Sg the order of the $7/2^+[613]$ and $9/2^-[734]$ states changes and the $9/2^-[734]$ state can become the isomer. In $^{257}$Rf the isomer state at 0.07 MeV has been found [EPJA 43, 175 (2010)] and tentatively assigned to the $11/2^-[725]$ state. However, all our calculations result the $11/2^-[725]$ state at energy larger then 0.3 MeV and with increasing $Z$ there is no tendency for lowering the energy of this state below 0.1 MeV. The HB approach results the $11/2^-[725]$ states at energies above 0.6 MeV.
Using the one-quasiparticle spectra calculated with the TCSM for $^{257}$Rf and $^{253}$No, the possible $\alpha$ decay scheme of $^{257}$Rf is suggested. The calculated $Q_\alpha$ values are consistent with the experimental values listed.
The change of the deformation in isotone chain destroys the smooth dependence of the one-quasiparticle energies on $Z$. 

$\beta_2 = 0.26$ 
$\beta_4 = -0.01$
Summary

- The energies of almost all low-lying one-quasiparticle states change rather smoothly in the isotone chains if there is no cross of the proton sub-shell, i.e. if the ground-state deformations of the isotones are close.

- The change of the deformation due to the proton shell effect (N=155 isotones) causes the rearrangement of the order of the one-quasiparticle states.
The simultaneous good agreement of the calculated and experimental spectra for the $N=149$ and 151 nuclei can not be achieved without strong variation of the parameters.

The calculations were performed with the TCSM and QPM which belong to the microscopic-macroscopic approach and with the self-consistent HB approach. All approaches qualitatively lead to the same conclusions.

The used simple shape parametrization is suitable to describe some properties of heavy nuclei.
$Q_\alpha = \begin{array}{ccccccccccccc}
6.7 & 7.6 & 8.4 & 8.6 & 9.8 & 9.4 & 9.9 & 10.7 & 11.4 & 10.6
\end{array}$
$^{64}$Ni + $^{207}$Pb $\rightarrow ^{270}$Ds + 1n

ground state: $E_{CN}^* \approx 14$ MeV

2qp isomer state: $E_{CN}^* \approx 14 - E_\mu \approx 12.8$ MeV

$W_{\text{sur}}(\text{isomer})/W_{\text{sur}}(\text{gs}) \approx 2$

For $E_\mu \approx 1.2$ MeV, the population of isomer state is $\approx \exp\left(-\frac{(E_\mu - E_{\text{rot}})}{T}\right) \approx 0.32$.

$\sigma_{ER}(\text{gs})/\sigma_{ER}(\text{isomer}) \approx 0.68:0.64$

exp.: $\approx 1:1$
1qp isomer state:

After the CN is cooled down by the neutron emission till $E^* < 8$ MeV

$$p_{is} \approx \exp(-E_{is}/T)/[1+\exp(-E_{is}/T)]$$

$T \approx 0.6$ MeV and $p_{is} > 0.35$ in the reaction treated.

The population of isomer state is quite probable.
$^{248}$Fm

$\beta_2$—solid lines, $\beta_4$—dashed lines

\[ \beta = 1.28 \]

\[ \lambda = 1.18 \]
$^{248}\text{Fm}$

- $\lambda = 1.18, \beta = 1.04$
- $\lambda = 1.30, \beta = 1.28$
- $\lambda = 0.9, \beta = 0.7$
- $\lambda = 1.04, \beta = 1.28$
\[ H = T + V(\rho, z) + V_{LS} + V_{L^2} \]

\[ \frac{1}{2} m \omega_z^2 (z - z_1)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, \ z < z_1 \]

\[ V(\rho, z) = \begin{cases} \frac{\epsilon}{2} m \omega_z^2 (z - z_i)^2 (1 + c_i (z - z_i) + d_i (z - z_i)^2) + \frac{1}{2} m \omega_\rho^2 \rho^2, & \text{if} \ z_1 < z < z_2 \end{cases} \]

\[ \frac{1}{2} m \omega_z^2 (z - z_2)^2 + \frac{1}{2} m \omega_\rho^2 \rho^2, \ z > z_2 \]

\[ V_{LS} = -\frac{2 \hbar \kappa_i}{m \omega_{0i}} (\nabla V \times \vec{p}) \hat{s} \]

\[ V_{L^2} = -\frac{\hbar \kappa_i \mu_i}{m^2 \omega_{0i}^3} \hat{l}^2 + \hbar \kappa_i \mu_i \omega_{0i} N_1 (N_1 + 3)/2 \delta_{if} \]

\[ \omega_{0i} = 41 \text{MeV} / A_i^{1/3}, \ A_i = a_i b_i/1.22^3, \ c_i = -2/z_i, \ d_i = -2/z_i^2, \]

\[ \omega_\rho / \omega_z = a_i / b_i, \ z_2 - z_1 = 2R_0 \lambda - a_1 - a_2 \]
Parameters

$35 \leq N - Z \leq 56$

_for neutrons_

\[
\kappa_n = -0.076 + 0.0058(N - Z) - 6.53 \times 10^{-5}(N - Z)^2 + 0.002 A^{1/3},
\]

\[
\mu_n = 1.598 - 0.0295(N - Z) + 3.036 \times 10^{-4}(N - Z)^2 - 0.095 A^{1/3},
\]

_for protons_

\[
\kappa_p = 0.0383 + 0.00137(N - Z) - 1.22 \times 10^{-5}(N - Z)^2 - 0.003 A^{1/3},
\]

\[
\mu_p = 0.335 + 0.01(N - Z) - 9.367 \times 10^{-5}(N - Z)^2 + 0.003 A^{1/3},
\]

The parts in front of the terms with $A^{1/3}$ vary:

(0.05-0.053) for $\kappa_n$, (0.075-0.0768) for $\kappa_p$, (0.88-0.92) for $\mu_n$, (0.58-0.61) for $\mu_p$. 

Strength parameters of paring interaction

\[ G_n = \left( 19.2 \pm 7.4 \frac{N-Z}{A} \right) A^{-1} \text{MeV} \]

\[ G_p \approx 0.075 \text{MeV}, \; G_p \approx 0.085 \text{MeV} \]

\[ A \approx 250 \rightarrow G_n \approx 0.075 \text{MeV}, \; G_p \approx 0.085 \text{MeV} \]

One-quasiparticle excitations

\[ E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} - \sqrt{(e_\mu' - e_F)^2 + \Delta^2} \]

Two-quasiparticle excitations

\[ E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} + \sqrt{(e_\mu' - e_F)^2 + \Delta^2} \]

\[ \Delta \geq 0.35 \text{MeV} \rightarrow \text{BCS approximation} \]
$^{270}\text{Hs}$

$\beta_2 = 0.25 \quad 0.262 \quad 0.256$

$\beta_4 = -0.026 \quad -0.006 \quad 0.026$